Preliminary Calculations of the Power Required to lift a Climber on the Space Elevator Under Conditions of Constant Velocity or Constant Power

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The following analysis models the climber as an object supported by a pure force. It neglects to take into account the rotary moment of inertia of the drive train, or any bearing or rolling resistance losses which will eat up some of the available power.

The first page is devoted to various constants needed in the analysis. The first calculation looks at the power required to lift the climber at constant velocity. Instantaneous acceleration is assumed. I am using this document as a scratch pad to organize my analysis of the design of a construction ribbon climber.

$R_e := 6.378 \times 10^6 \cdot m$ Radius of the Earth (altitude = 0)

$G := 6.67 \cdot 10^{-11} \cdot \frac{m^3}{sec^2 \cdot kg}$ Newton's gravitational constant

$M_e := 5.9788 \cdot 10^{24} \cdot kg$ Mass of the Earth

$\omega := 7.2929 \cdot \frac{10^{-5}}{sec}$ Angular speed of Earth

$r := R_e, R_e + 100 \cdot 10^3 \cdot m .. R_e + 10^8 \cdot m$ Range variable for altitude from center of earth

$a_c(r) := \frac{M_e \cdot G}{r^2} - r \cdot \omega^2$ Acceleration felt by ribbon climber as function of altitude

$a_c(R_e) = 9.769 \frac{m}{s^2}$ g at equator (accounting for earth's spin)

$$g := \frac{M_e \cdot G}{R_e^{\,2}}$$

g at poles where there is no centripetal force

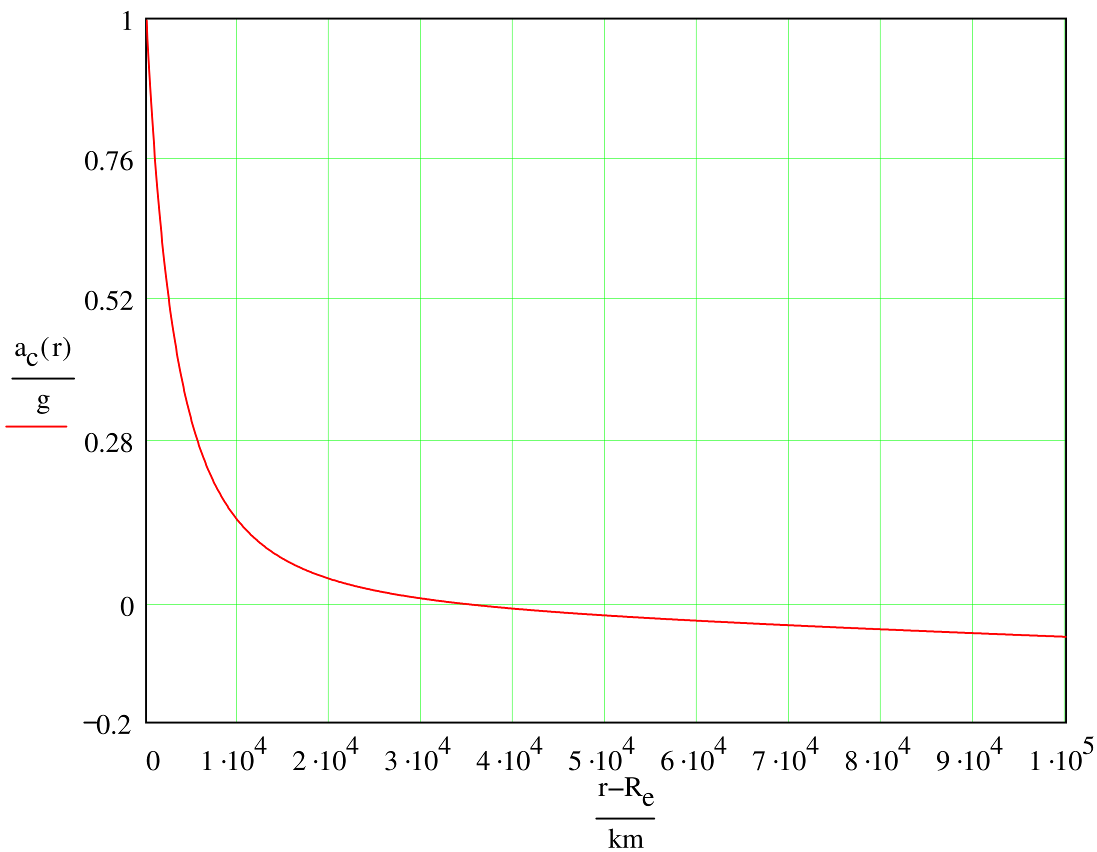
$$g = 9.803 \frac{m}{s^2}$$

Common value for g

$$\frac{g - a_c\left(R_e\right)}{g} \cdot 100 = 0.346$$

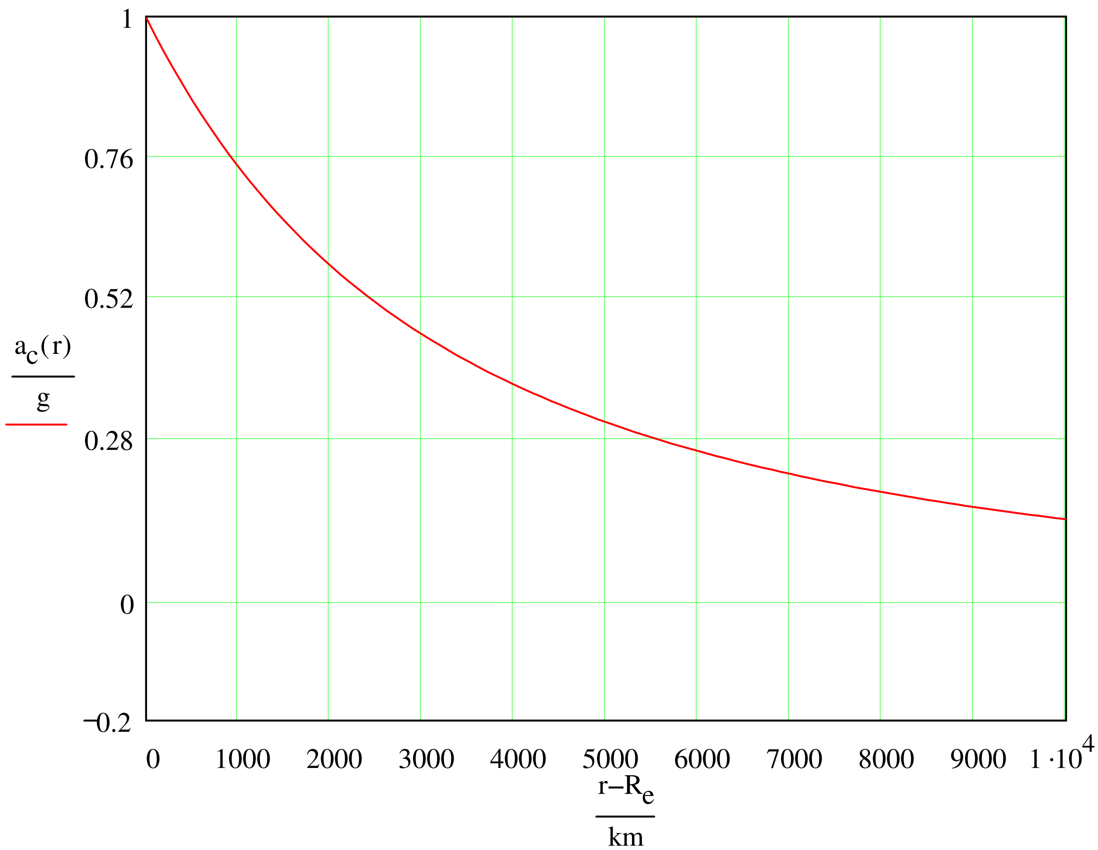
Percentage difference in weight between pole and equator

Graph of acceleration felt by climber (ratio with earth normal gravity) vs altitude (kilometers) from Earth's surface at the Equator



The acceleration is expressed as a ratio of the acceleration to earth normal gravity at the poles.

Detail view of above graph of acceleration felt by climber (ratio with earth normal gravity) vs altitude (kilometers) from Earth's surface at the Equator (curve stopped at 10^4 km up)



$$a_c\left(R_e + 10^8 \cdot m\right) = -0.531 \frac{m}{s^2}$$

$$\frac{a_c\left(R_e + 10^8 \cdot m\right)}{g} = -0.054$$ Maximum negative acceleration at end of ribbon

Finding the altitude where the acceleration is zero:

$r := R_e + 4 \cdot 10^7 \cdot m$ Guess value for root of function

$\text{root}\left(a_c(r), r\right) = 4.217 \times 10^{7}\,\text{m}$ Root finding function to see where a=0

$\text{Alt} := \text{root}\left(a_c(r), r\right) - R_e$

$\text{Alt} = 3.579 \times 10^{4}\,\text{km}$ This is the altitude of Geosynchronous orbit

Some example values of the ratio of climber acceleration to g:

$r := R_e + 10^{5} \cdot \text{m}$ Altitude of 100 kilometers

$a_c(r) = 9.468\,\frac{\text{m}}{\text{s}^2}$ Climber acceleration

$\frac{a_c(r)}{g} = 0.966$

$r := R_e + 10^{6} \cdot \text{m}$ Altitude of 1,000 kilometers

$a_c(r) = 7.287\,\frac{\text{m}}{\text{s}^2}$ Climber acceleration

$\frac{a_c(r)}{g} = 0.743$

$r := R_e + 10^{7} \cdot \text{m}$ Altitude of 10,000 kilometers

$a_c(r) = 1.4\,\frac{\text{m}}{\text{s}^2}$ Climber acceleration

$\frac{a_c(r)}{g} = 0.143$

Now to calculate the power required to lift the climber at the design speed

$r := R_e, R_e + 100 \cdot 10^3 \cdot m .. R_e + 10^8 \cdot m$

Range variable for altitude from center of earth

$m_c := 900 \cdot kg$

The mass of the first climber is 900 kg.

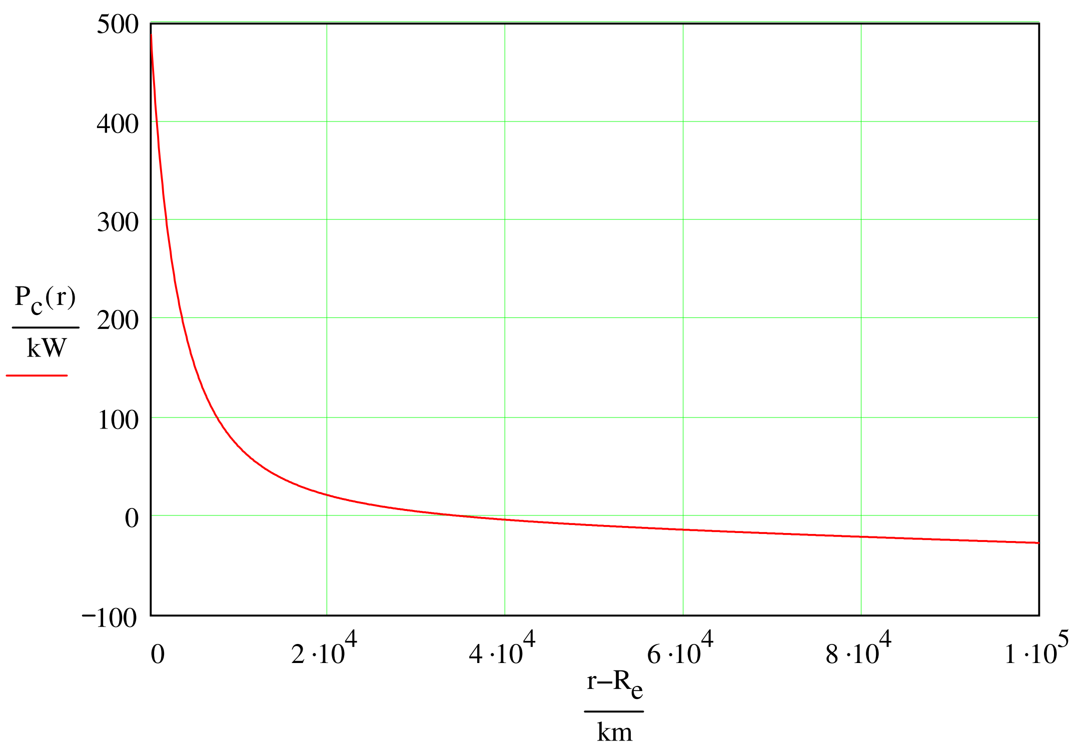
$v_c := 200 \cdot \frac{km}{hr}$

The original design velocity of the climber

$P_c(r) := m_c \cdot a_c(r) \cdot v_c$

Power required to raise the climber

Graph of Power (kiloWatts) vs Altitude up the ribbon (kilometers) for Constant Speed case



Find power required at surface of Earth:

$P_c(R_e) = 488.467\,kW$ $P_c(R_e) = 655.045\,hp$ Power required at earth level

At what altitude is the power required equal to or less than 100 kW?

$r := R_e + 1 \cdot 10^4 \cdot km$ guess value

Given

$100 \cdot kW = m_c \cdot a_c(r) \cdot v_c$

$z := Find(r)$ $z = 1.387 \times 10^4 km$

$Alt_{100kW} := z - R_e$

$Alt_{100kW} = 7.489 \times 10^3 km$

The altitude above which the power required is less than 100 kW is ~7500 km.

To find the total energy required for the trip to ribbon end:

$r := R_e, R_e + 100 \cdot 10^3 \cdot m .. R_e + 10^8 \cdot m$ Range variable for altitude from center of earth

$$a_c(r) := \frac{M_e \cdot G}{r^2} - r \cdot \omega^2$$

$$E_t := \int_{R_e}^{R_e+Alt} m_c \cdot a_c(r)\, dr$$

$E_t = 1.211 \times 10^4 kW \cdot hr$ Total Energy required to get to GEO

The average energy my house uses in a month is 1500 kW-hrs. How many months of my house energy is required?

$month := 30 \cdot day$ Defining a month for MathCAD

$$\frac{E_t}{1500 \cdot kW \cdot \frac{hr}{month}} = 8.075\,month$$

The time required to get to GEO (no acceleration time included):

$$t := \frac{Alt}{v_c}$$

$$t = 7.456\,day$$

Eight months of the energy used by my house must be applied within 7.5 days

The energy you get from out past GEO (which must be dissipated in the braking system somehow):

$$E_{out} := \int_{R_e+Alt}^{R_e+10^5 km} m_c \cdot a_c(r)\,dr$$

$E_{out} = -4.914 \times 10^3\,kW \cdot hr$ Energy to be dissipated from GEO to ribbon end

How long does the trip to the end of the ribbon take at constant speed?

$$T_{trip} := \frac{10^5 \cdot km}{200 \cdot \frac{km}{hr}}$$

$T_{trip} = 20.833\,\text{day}$ Time to end of ribbon assuming instantaneous acceleration and deceleration

Assume that the power is fixed at 100 kW and calculate the speed of the climber for constant power running

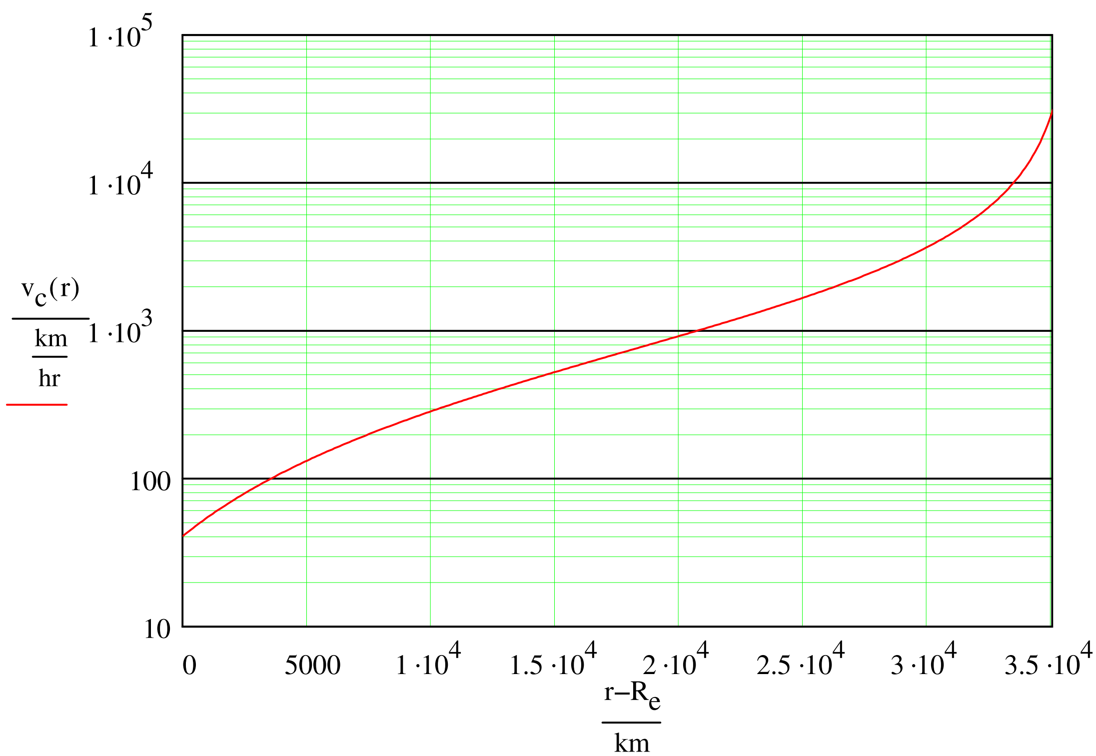
The height range variable stops before GEO because with constant power, as the force dragging the climber down decreases, the velocity increases to infinity. It is not physically possible for a climber to roll on the ribbon at some speed beyond a few hundred km/hr because of stress limitations in the wheels and acceleration limits imposed by the safety factor on the tensile strength of the ribbon. This graph just shows what is theoretical without reference to physical limits.

$$r := R_e, R_e + 100 \cdot km .. R_e + 35000km$$

$$P := 100 \cdot kW$$

$$v_c(r) := \frac{P}{m_c \cdot a_c(r)}$$

Graph of possible speed of climber vs height up ribbon for constant power (100kW)



Find the altitude at which the climber hits 200 km/hr

$r := R_e + 7000km$ Guess value to seed the find function

Given

$$200\frac{km}{hr} = \frac{P}{m_c \cdot a_c(r)}$$

$q := Find(r)$ Temporary variable to find the height

$q = 1.387 \times 10^4 km$

$Alt_{maxv} := q - R_e$

$Alt_{maxv} = 7.489 \times 10^3 km$

Above ~7500 km the climber hits 200 km/hr and as long as the power doesn' t fall off faster, the speed could be higher.

During a solar storm, the radiation belt extends out five times the radius of the earth from just above the surface. We want to make it through this belt as fast as we can.

$R_e \cdot 5 = 3.189 \times 10^4 km$ Just short of GEO

$Alt - R_e \cdot 5 = 3.9 \times 10^3 km$

What is the initial possible speed of the climber? (Again, no acceleration limits applied.)

$r := R_e, R_e + 100 \cdot 10^3 \cdot m .. R_e + 33000km$ Same range variable for height

$$a_c(r) := \frac{M_e \cdot G}{r^2} - r \cdot \omega^2$$

$$v_c(r) := \frac{P}{m_c \cdot a_c(r)} \qquad v_c(R_e) = 40.944 \frac{km}{hr}$$

Assume the trip starts with zero velocity at the Earth's surface. What constant acceleration is required until the climber gets to 200 km/hr at 7500 km up?

$d := Alt_{100kW}$ Renaming the variable for the height up the ribbon to 200 km/hı

$$d = 7.489 \times 10^3 km$$

$$v_{max} := 200 \cdot \frac{km}{hr}$$

$a_{ave} := \frac{1}{2} \cdot \frac{v_{max}^2}{d}$ Equation for average acceleration given distance and final v

$$a_{ave} = 2.061 \times 10^{-4} \frac{m}{s^2}$$

$\frac{a_{ave}}{g} = 2.102 \times 10^{-5}$ This acceleration eats up very little factor of safety in ribbon strength

The time it takes to get to 7500 km up with constant acceleration of a_{ave} is:

$$T_{acons} := \frac{v_{max}}{a_{ave}} \qquad T_{acons} = 3.121 \, day$$

The time it takes to go 7500 km at 200 km/hr constant speed is:

$$T_{vcons} := \frac{d}{v_{max}}$$

$$T_{vcons} = 1.56\,day$$

Time added to trip to GEO from acceleration time:

$$\Delta t := T_{acons} - T_{vcons}$$

$$\Delta t = 1.56\,day$$

Total time to GEO at constant velocity (no accel): $t = 7.456\,day$

Time at 200km/hr out to GEO after acceleration:

$$t_2 := \frac{Alt - d}{v_{max}} \qquad t_2 = 5.896\,day$$

Total time to GEO with acceleration: $T_{tot1} := T_{acons} + t_2$

$$T_{tot1} = 9.016\,day$$

Calculate the possible acceleration from the constant power curve of velocity:

The range variable for height up the ribbon from the center of earth in this calculation stops before GEO because the possible acceleration equation has a singularity at GEO.

$$r := R_e, R_e + 100\cdot km .. R_e + 35000\cdot km$$

$P := 100 \cdot kW$ Constant power

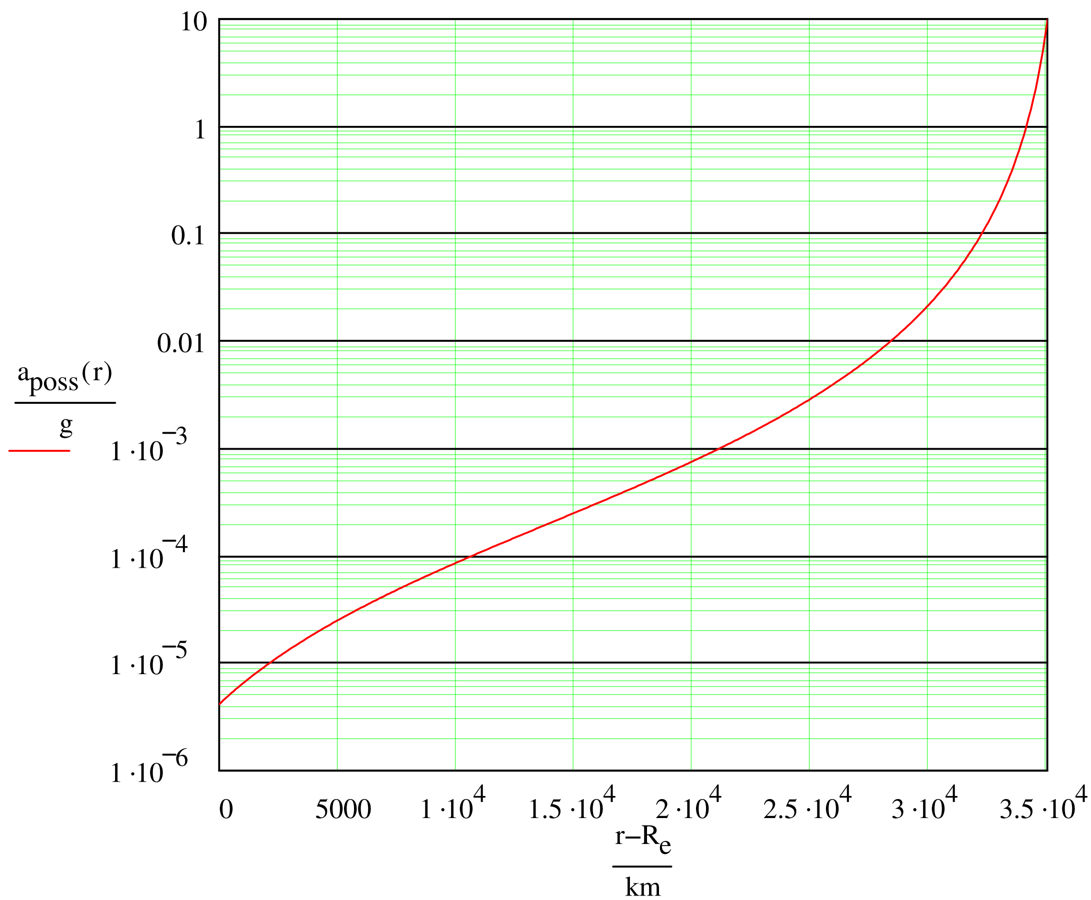
$$a_c(r) := \frac{M_e \cdot G}{r^2} - r \cdot \omega^2$$ Acceleration dragging on climber

$$v_c(r) := \frac{P}{m_c \cdot a_c(r)}$$

The possible acceleration of the climber under constant power is determined from the relation a=v*dv/dx.

$$a_{poss}(r) := v_c(r) \cdot \frac{d}{dr} v_c(r)$$

Graph of possible acceleration of climber (ratio with g) vs height up ribbon (km) for constant power (100kW)



The average acceleration calculated above (2.102e-5 ratio) is higher than the lowest acceleration ratio shown here. As shown below, the average constant acceleration as previously calculated causes the power required to exceed 100 kW. I need to calculate the power required as a function of time (instead of as a function of height up ribbon like all previous calculations) for constant acceleration case.

Calculating the power required for the first three days of climbing at constant acceleration:

$t := 0 \cdot sec, 10 \cdot sec .. T_{acons}$ range variable for time to 7500 km up

$h(t) := \left(\frac{1}{2} \cdot a_{ave} \cdot t^2\right) + R_e$ Height up ribbon now a function of time

$V(t) := a_{ave} \cdot t$ Initial velocity is zero

$a_c(t) := \frac{M_e \cdot G}{(h(t))^2} - h(t) \cdot \omega^2$ Drag acceleration expressed as f(t)

$A_{tot}(t) := a_c(t) + a_{ave}$ Drag force plus inertial force to lift climber

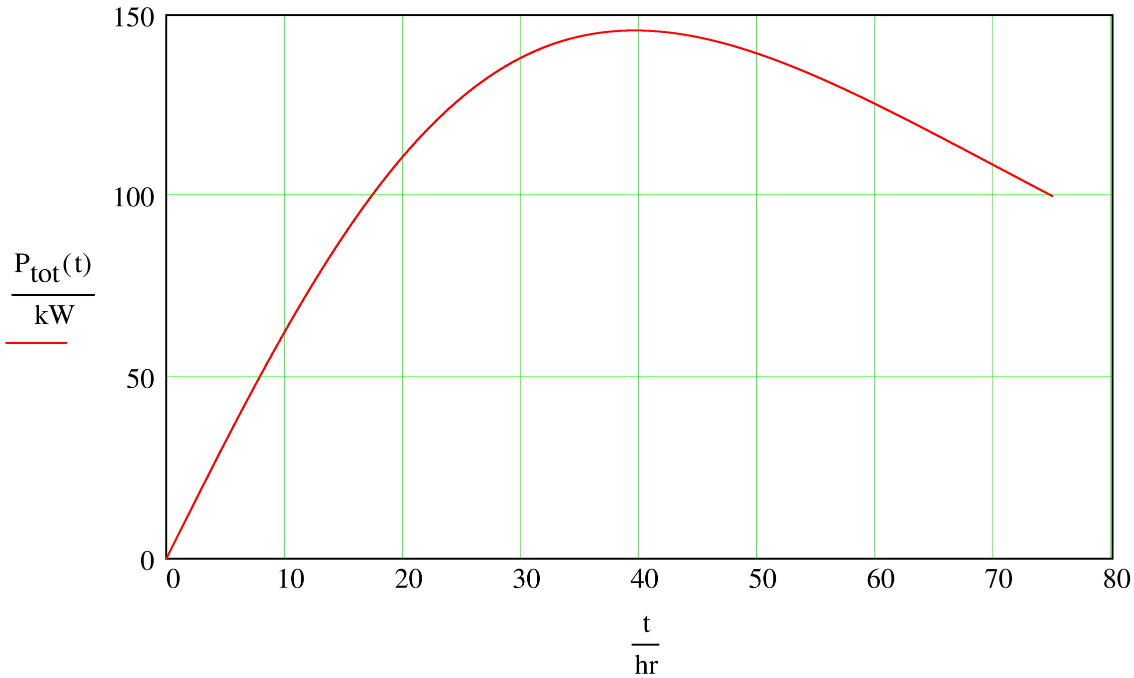
$P_{tot}(t) := m_c \cdot A_{tot}(t) \cdot V(t)$ Power required to overcome drag and accelerate

I am examining the case of starting the trip up the ribbon with constant acceleration because it is one of the easiest to deal with and conceptually common. Another assumption in this calculation that is not physically realizable with a real ribbon and climber is the step function constant acceleration that I' m using here. The step function of acceleration has a singularity in its derivative at the instant of the beginning of acceleration and the instant of the acceleration ending (the point of constant velocity). This third derivative of position is called "jerk" and in a real climber, acceleration must also follow a curve that starts at zero at time zero and smoothly increases to the desired value.

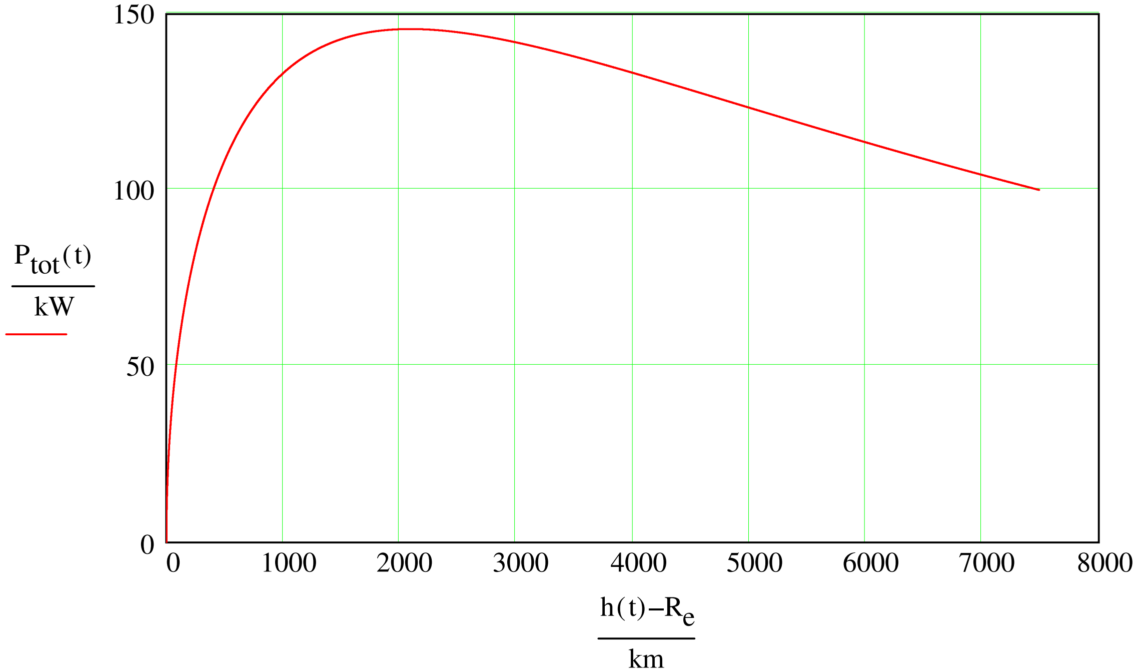
Each of these calculations adds a level of complexity not seen in the previous one and the pattern I' m seeing is that each bit of added reality is making the trip out to GEO take longer and longer. I haven' t even added the effect of the mass moment of inertia of the drive train, which will also slow the climb down. It takes power to accelerate the wheels up to speed.

I have not figured out yet how to deal with the trip from GEO to the end of the ribbon. Clearly the climber must decelerate before it gets to the end of the ribbon. It must also brake the increasing acceleration due to the centrepital force.

Graph of power (kW) required to lift climber at constant acceleration vs time (hrs) up ribbon to 7500 km



Graph of power (kW) required to lift climber at constant acceleration vs height (km) up ribbon to 7500 km



These two graphs above show that constant acceleration at the previously calculated rate causes the power required to be higher than 100 kW, and the problem occurs below 5000 km, just as would be predicted by the possible acceleration curve above. I will recalculate the acceleration taking longer to accelerate up to 200 km/hr to see how low the average acceleration has to be to keep the power at or below 100 kW.

The way to lower the acceleration will be to choose a higher altitude to reach 200 km/hr by multiplying the original 7500 km by a factor. I will manually tune the facto to get the desired result.

$f := 2.12$ Tunable factor to find new height to reach 200 km/hr

$d := f \cdot 7500 \cdot km$ Putting a multiplier into the old height to 200 km/hr

$d = 1.59 \times 10^4 km$

$v_{max} := 200 \cdot \frac{km}{hr}$ Desired maximum velocity of 200 km/hr

$$a_{ave} := \frac{1}{2} \cdot \frac{v_{max}^2}{d}$$ Finding a new, lower average acceleration

$$a_{ave} = 9.706 \times 10^{-5} \frac{m}{s^2}$$

$$\frac{a_{ave}}{g} = 9.9 \times 10^{-6}$$ This is significantly lower than the previous value

The time it takes to get to d km up at constant acceleration is:

$$T_{acons} := \frac{v_{max}}{a_{ave}}$$

$T_{acons} = 159\,hr$ $\quad$ $T_{acons} = 6.625\,day$

Time at 200km/hr out to GEO after acceleration:

$t_2 := \frac{Alt - d}{v_{max}}$ $\quad$ $t_2 = 4.144\,day$

Total time to GEO with new lower acceleration: $T_{tot2} := T_{acons} + t_2$

$T_{tot2} = 10.769\,day$ $\quad$ $T_{tot1} = 9.016\,day$ $\quad$ $T_{tot2} - T_{tot1} = 1.752\,day$

The new lower acceleration adds another day and three-quarters to the trip to GEO. Next I will show the graphs of power vs time and power vs altitude.

$t := 0 \cdot sec, 10 \cdot sec .. T_{acons}$ range variable for time to d km up

$$h(t) := \left(\frac{1}{2}\cdot a_{ave}\cdot t^2\right) + R_e$$

height up ribbon now a function of time

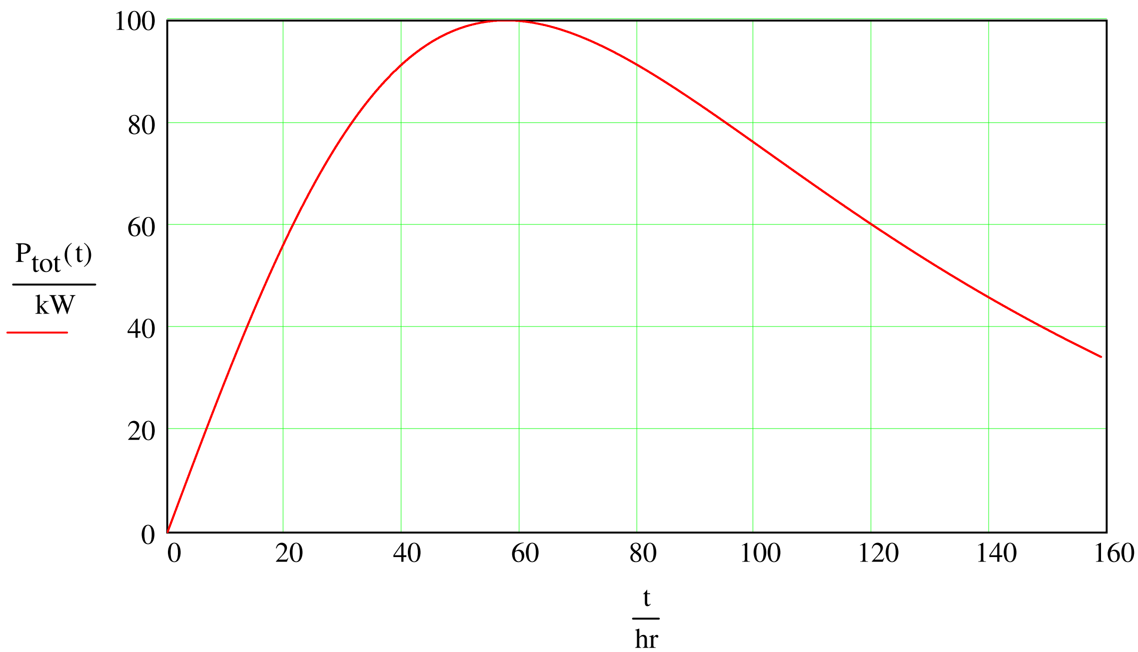
$$V(t) := a_{ave}\cdot t$$

$$a_c(t) := \frac{M_e\cdot G}{(h(t))^2} - h(t)\cdot\omega^2$$

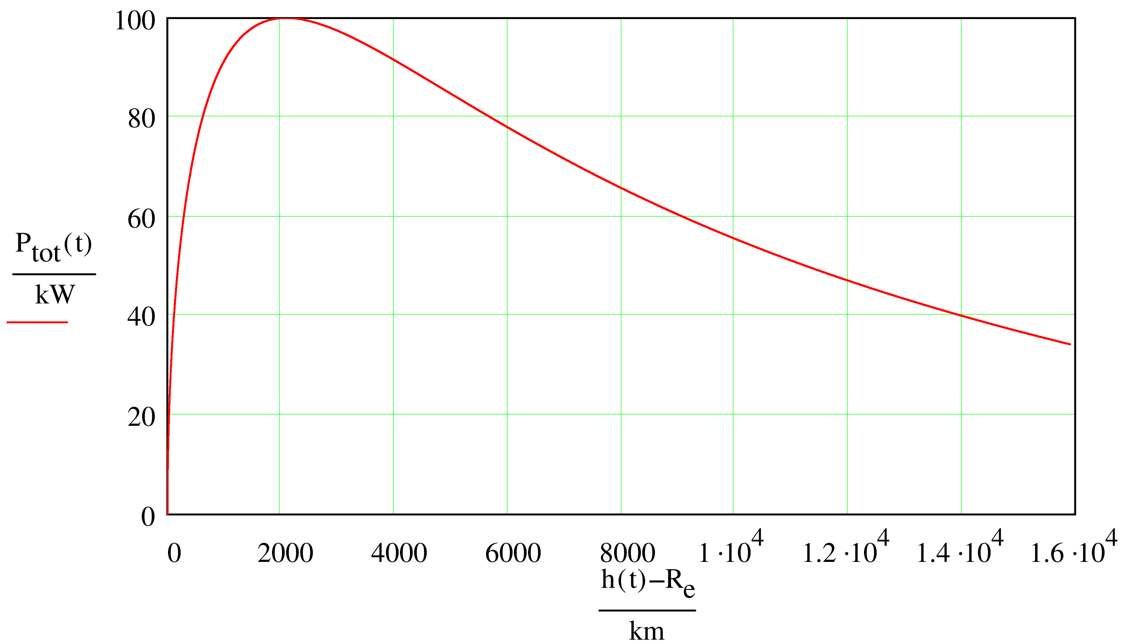
$$A_{tot}(t) := a_c(t) + a_{ave}$$

$$P_{tot}(t) := m_c\cdot A_{tot}(t)\cdot V(t)$$

Graph of power required to lift climber at constant acceleration vs time up ribbon, reaching 200 km/hr at a higher altitude (lower average acceleration)



Graph of power required to lift climber at constant acceleration vs height up ribbon, reaching 200 km/hr at a higher altitude (lower average acceleration)



$$P_{tot}(58 \cdot hr) = 99.997\,kW$$

This curve shows that once the climber peaks out at the 100 kW point, it could begin to accelerate more because the drag begins to drop off more quickly. You need to start at low acceleration given the 100 kW limitation, but the acceleration can be cranked up above 2000 km up the ribbon.

The initial acceleration is small enough that it cannot produce a problem for the safety factor of the ribbon. This may not be true in the higher acceleration region midway up the ribbon, or values possible from GEO out to the end of the ribbon. I need to develop an expression for the safety factor remaining in the tensile strength of the ribbon for a climber at any arbitrary altitude and acceleration. The way to see why acceleration is important is to remember how the cross-sectional area of the ribbon fibers was calculated using a safety factor of two. The ribbon calculation shown in the book starts with a climber not moving, just hanging on the ribbon at the surface of the earth. *If that climber were accelerating upward at one g, it would look twice as heavy to the ribbon and the safety factor on the tensile strength of the ribbon would be completely used up.*

Another thing these calculations have made abundantly clear is that constant power/variable speed running requires programming the motion of the climber with some kind of polynomial acceleration curve to achieve close to constant power running. (This might be handled by a feedback loop on the real climber, instead of an open loop controller.) The singularities in the curves indicate that perfect constant power running is not possible (at least in a system with no rotary mass moment of inertia.) I will have to expand on this work by developing a candidate climber wheel assembly and including the effect of the rotary mass moment of inertia.

Once I have a curve for the torque required to achieve a near-constant power running, I can then determine if the axial gap electric motors I am considering have this torque/speed capability. This will allow me to see how many stages of reduction/speed increasing are required in the transmission.

Stress calculations on the wheels will determine what is the maximum speed the climber can obtain.

I also want to look at how quickly the laser power will diminish with altitude to see if that will put additional limits on the speed and acceleration.

I am trying to write a complete system simulation model that will take into account all kinds of electrical and mechanical effects, but I need to do more preliminary work before attempting this.

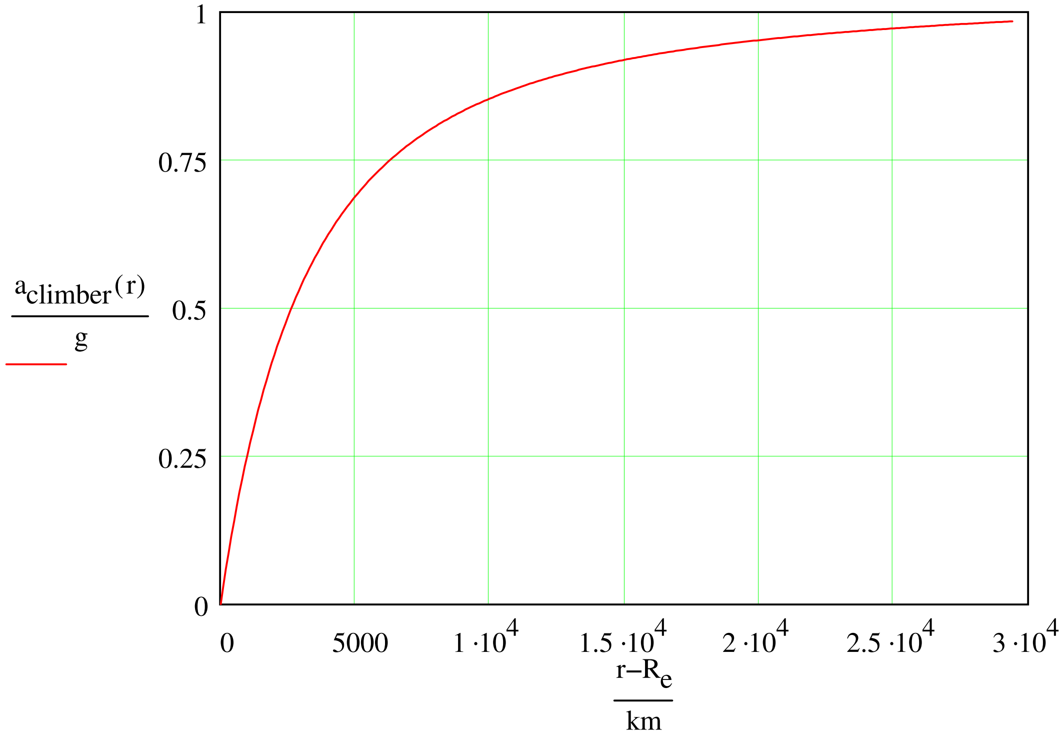
Looking at maximum acceleration from a ribbon structural point-of-view:

The ribbon cross-section is sized at every point to create a constant safety factor of 2 everywhere with a climber at the earth' s surface hanging on the ribbon but not moving. Also, by free-body diagrams it can be shown that the climber only affects the tension in the ribbon below it, not above it. The goal in this calculation is to calculate the allowed acceleration of the climber as gravity drops off with altitude, assuming that the combination of the weight and the acceleration must always equal the original weight of the climber with no acceleration. This will be compared with the possible acceleration calculated above, and with constant acceleration profiles to better refine the time to GEO. The speed limit of the climber will be increased to the maximum of electric powered vehicles on earth, 500 km/hr.

$r := R_e, R_e + 100\cdot km .. Alt$ $\quad$ $Alt = 3.579\times 10^4 km$ $\quad$ Altitude of GEO

$$a_{climber}(r) := \frac{G\cdot M_e}{R_e^2} - R_e\cdot\omega^2 - \left(\frac{G\cdot M_e}{r^2} - r\cdot\omega^2\right)$$

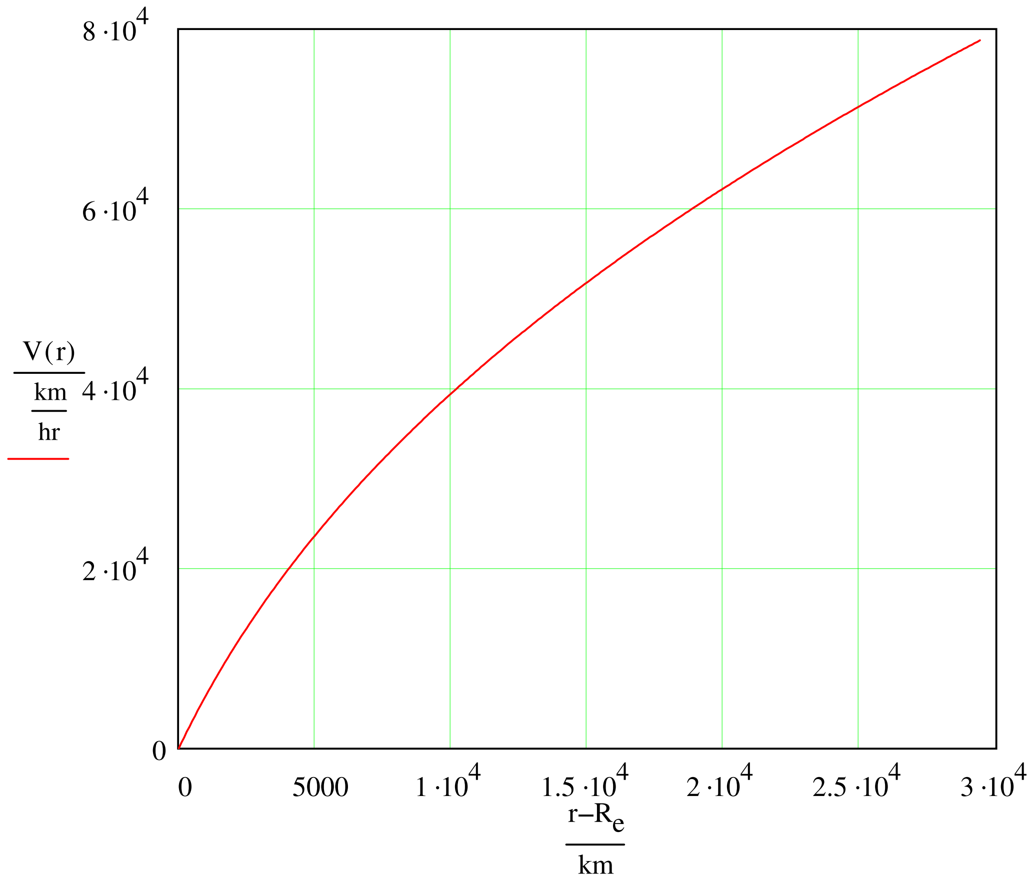
Graph of maximum acceleration of the climber as the drag due to gravity drops off with altitude



$$V(r) := \left(2\cdot\int_{R_e}^{r} a_{climber}(r)\,dr\right)^{\frac{1}{2}}$$

velocity of the climber as a function of the new acceleration curve

Graph of the velocity of the climber accelerating by the maximum acceleration curve as the drag due to gravity drops off with altitude



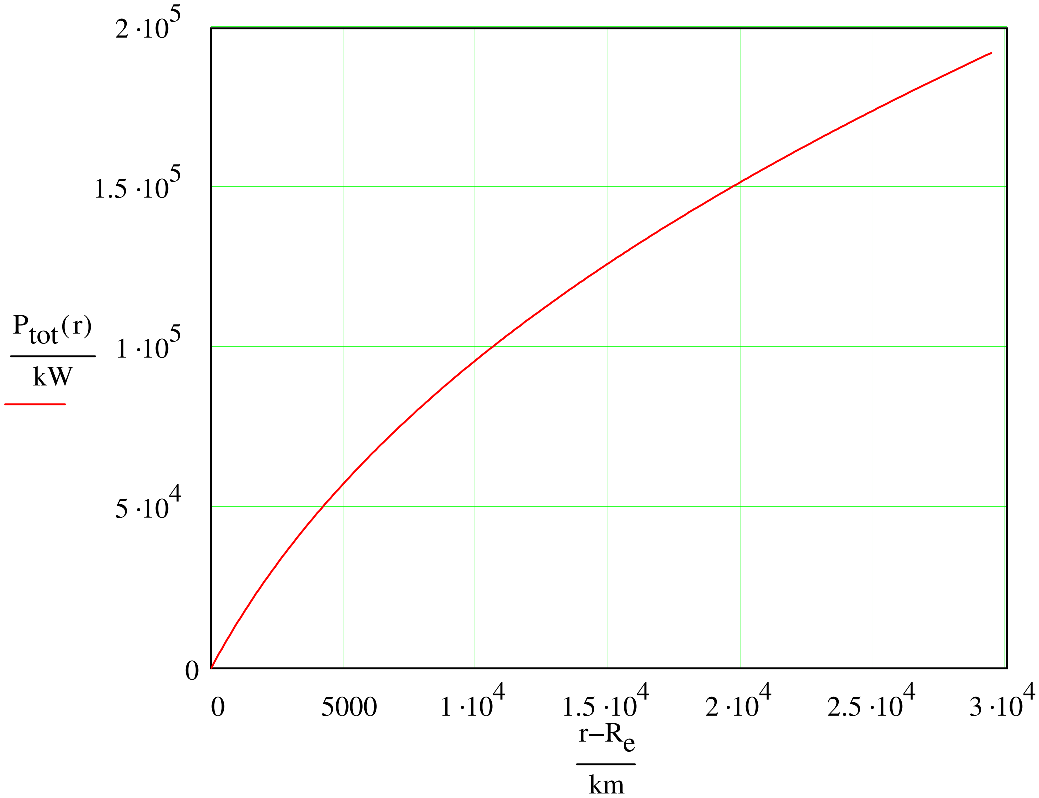
We see from this graph that the maximum acceleration curve very quickly accelerates the climber past any practical velocity.

$$a_c(r) := \frac{M_e \cdot G}{r^2} - r \cdot \omega^2$$ Drag acceleration expressed as f(r)

$$A_{tot}(r) := a_c(r) + a_{climber}(r)$$ Drag force plus inertial force to lift climber

$$P_{tot}(r) := m_c \cdot A_{tot}(r) \cdot V(r)$$ Power required to overcome drag and accelerate by the maximum allowed acceleration curve above

Graph of the power required to accelerate the climber by the maximum acceleration curve as the drag due to gravity drops off with altitude

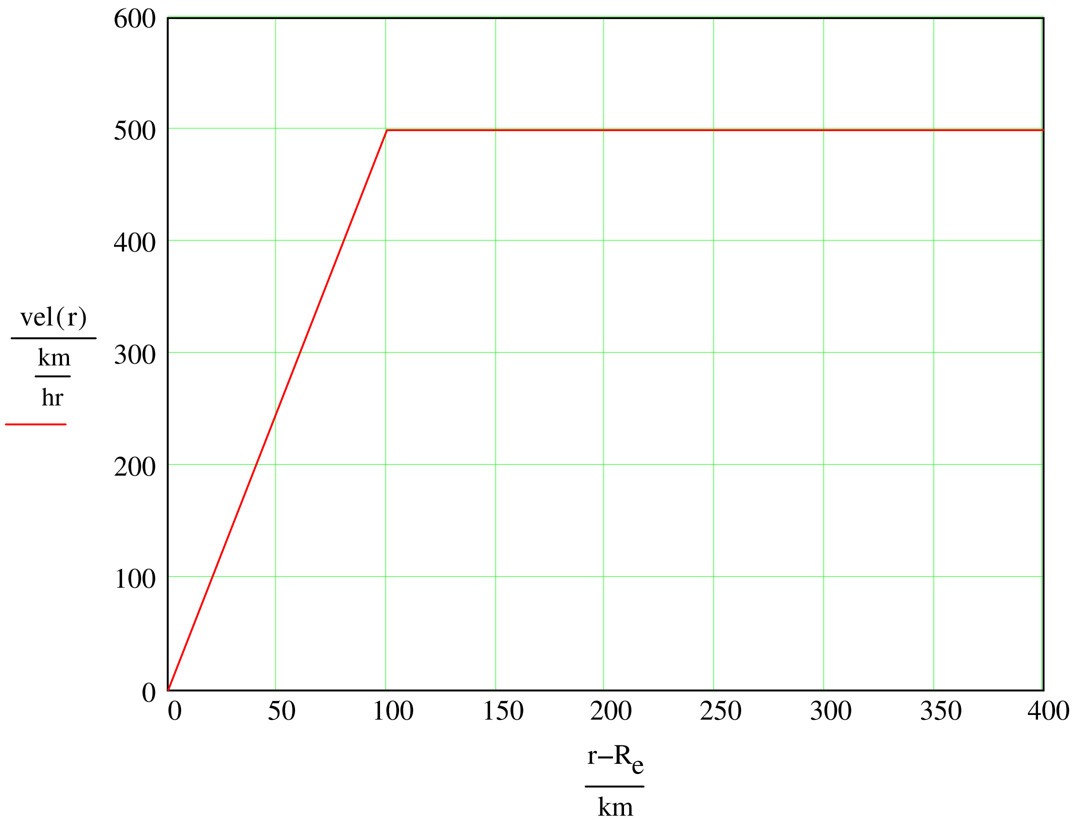


This graph shows that the limitation on acceleration comes from the available laser power, not the allowed acceleration due to the safety factor on the stress in the ribbon Since the power available is nowhere near that required to accelerate the climber by the maximum acceleration allowed by the ribbon, then the climber cannot accelerate a that rate.

If the velocity is limited to 500 km/hr, then

$$\mathrm{vel}(r) := \begin{cases} V(r) & \text{if } V(r) \le 500 \cdot \dfrac{km}{hr} \\ \left(500 \cdot \dfrac{km}{hr}\right) & \text{otherwise} \end{cases}$$

Graph of the velocity of the climber accelerating by the maximum acceleration curve, but limited to a maximum speed of 500 km/hr



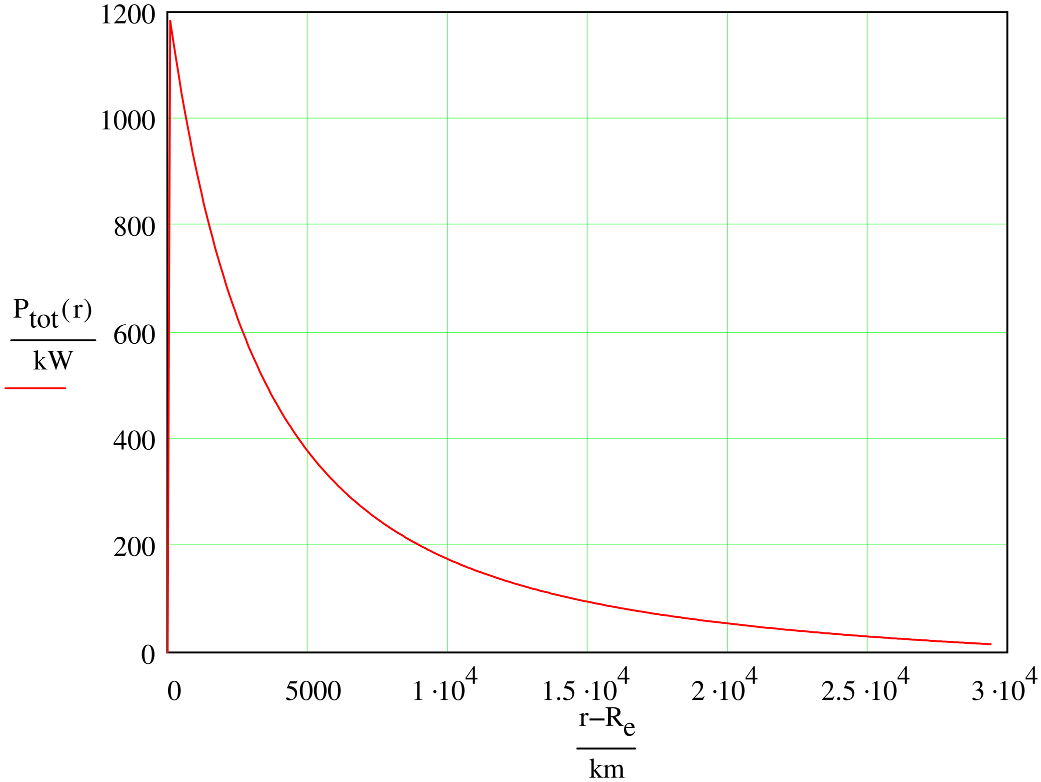
Once the speed hits maximum, the acceleration goes to zero. Only the gravitational drag remains.

$$A_{tot}(r) := \begin{cases} a_c(r) + a_{climber}(r) & \text{if } vel(r) < 500 \cdot \frac{km}{hr} \\ a_c(r) & \text{otherwise} \end{cases}$$

$$a_{climber}\left(R_e + 100 \cdot km\right) = 0.301\,\frac{m}{s^2}$$ Peak acceleration just before maximum speed

$$P_{tot}(r) := m_c \cdot A_{tot}(r) \cdot vel(r)$$ Power required to overcome drag and accelerate

Graph of the Power Required for Maximum Allowed Acceleration of the Climber up to 500 km/hr



It is clear that what limits the speed of the climber is not the maximum acceleration given by the safety factor of the ribbon, but the power available from the laser. A megawatt laser will be available for the cargo climbers. If the small construction climbers could handle the power, they could accelerate at the maximum rate allowed by the ribbon.

Conclusion:

If the power to the climbers is limited to the 100 kW range, the time to GEO is much longer than it would be if the climbers had enough power to accelerate at the maximum rate allowed by the safety factor of the ribbon.

Detail of the Graph of the Power Required for Maximum Allowed Acceleration of the Climber up to 500 km/hr